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Fermion Coherence Hamiltonians

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Abstract

We have established that the most general form of Hamiltonian that preserves fermionic coherent states stable in time, is that of the nonstationary free fermionic oscillator. This is to be compared with the earlier result of boson coherence Hamiltonian, which is of the more general form of the nonstationary forced bosonic oscillator. If however one admits Grassmann variables as Hamiltonian parameters then the coherence Hamiltonian takes again the form of (Grassmannian fermionic) forced oscillator.

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1 Introduction

The time evolution of coherent states (CS) has attracted a great deal of attention since the introduction of Glauber CS of the harmonic oscillator [9, 10]. These CS can be defined as eigenstates of the photon (boson) annihilation operator a . They form an overcomplete set, providing a very useful continuous representation in the Hilbert space of states (for details see e.g. [17] and references therein). Of particular interest has been the determination of the general

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form of Hamiltonian for which an initial CS remains coherent under time evolution. It was established that this general form is that of the nonstationary (boson) forced oscillator Hamiltonian [11, 23, 24]

$$H_{\text{cs}} = \omega(t)a^\dagger a + f(t)a^\dagger + f^*(t)a + g(t), \quad (1)$$

where $\omega(t)$ and $g(t)$ are arbitrary time-dependent real functions, and $f(t)$ is arbitrary time-dependent complex function. The time evolution $|z; t\rangle$ of an initial CS $|z\rangle$, $z \in C$, governed by the Hamiltonian (1) remains, for all later times, eigenstate of the photon annihilation operator a . This eigenvalue property of Glauber CS allows easy calculation of means of normally ordered operators, in particular of the photon (boson) number operator. The Hamiltonian (1) will be referred to as *boson canonical coherence preserving Hamiltonian*, or shortly *boson coherence Hamiltonian*. Coherence Hamiltonians for SU(2) and SU(1,1) group related CS are found in [8, 6].

CS for fermion systems are defined in analogy to the canonical boson CS [16, 18, 17, 1, 19, 20, 14, 13, 3]. The overcompleteness property of the set of fermion annihilation operator eigenstates has been proved in [14, 13] using the Berezin integration rules for Grassmann variables. Extension of canonical CS to the case of pseudo-Hermitian fermions was performed in [4].

Eigenstates of fermion annihilation operators have been previously considered by Schwinger [25] and Martin [22] who noted that, since fermion ladder operators anticommute their eigenvalues are not ordinary numbers (they are Grassmann variables instead). Nevertheless many of the mathematical properties of Glauber CS and related methods of analysis of statistical properties of boson fields have their formal counterparts for Fermi fields [3]. However, the important problem of coherence preserving fermionic Hamiltonians was so far not considered in the literature. And our purpose in the present article is to establish the most general form of Hamiltonians, which preserve the fermionic CS stable under the time evolution.

The organization of the article is as follows. We start with a brief review in Sec. II of time evolution and temporal stability of canonical boson CS. In Sec. III we study the temporal stability of fermionic CS and we show, by using the fermionic analog of the invariant boson ladder operator method [21, 12, 26] that the most general form of Hamiltonian that preserves fermionic CS stable in time is in the form of free (nonforced) fermionic oscillator. In the last section we consider the evolution of fermion CS governed by the Grassmannian Hamiltonians and show that the Grassmannian forced fermionic oscillator also preserves the temporal stability of CS. The paper ends with concluding remarks.

2 Canonical boson CS and their temporal stability

The standard boson CS (called also Glauber CS, or *canonical* CS) are defined as the right eigenstates of the boson (photon) annihilation operator a [9, 10]

$$a|z\rangle = z|z\rangle, \quad (2)$$

the eigenvalue z being a complex number. The annihilation and creation operators a and a^\dagger satisfy the boson commutation relations $[a, a^\dagger] = aa^\dagger - a^\dagger a = 1$. The normalized CS $|z\rangle$ can be constructed in the form of displaced ground state $|0\rangle$ [9, 10],

$$|z\rangle = D(z)|0\rangle, \quad D(z) = e^{za^\dagger - z^*a}, \quad (3)$$

and their expansion in terms of the number states $|n\rangle$ reads

$$|z\rangle = e^{-\frac{|z|^2}{2}} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} |n\rangle. \quad (4)$$

The problem of temporal stability of canonical boson CS is solved by Glauber [11] and Mehta and Sudarshan [23] (in the case of one mode CS, and for n -mode CS - by Mehta et al. [24]). The result is that the most general Hamiltonian that preserves an initial CS $|z\rangle$ stable in later time is of the form of the nonstationary forced oscillator Hamiltonian H_{cs} , eq. (1). The Hamiltonian (1) that preserves CS (3) stable is shortly called *coherence Hamiltonian*. Thus the boson coherence Hamiltonian takes the form of a *non-stationary forced oscillator* Hamiltonian. Here "stable" means that the time evolved state $|z; t\rangle$, $i\hbar\partial_t|z; t\rangle = H_{cs}|z; t\rangle$, remains eigenstate of a , possibly with a time-dependent eigenvalue $z(t)$,

$$a|z; t\rangle = z(t)|z; t\rangle \quad (5)$$

From the latter equation one deduces that, up to a time-dependent phase factor $\exp(i\varphi(t))$, the time-evolved CS $|z; t\rangle$ depends on time t through $z(t)$, that is

$$|z; t\rangle = e^{i\varphi(t)} |z(t)\rangle, \quad |z(t)\rangle = e^{a^\dagger z(t) - z^*(t)a} |0\rangle \quad (6)$$

One says that for boson system with Hamiltonian (1) an initial canonical CS remains CS all the later time [11, 23] (or remains *temporally stable*). For the Hamiltonian system (1) the time dependent eigenvalue value $z(t)$ obeys the equation [11, 23]

$$i\dot{z} = \omega(t)z + f(t) \quad (7)$$

the solution of which takes the explicit form ($z = z(0)$)

$$z(t) = \tilde{\beta}(t)z + \tilde{\gamma}(t), \quad \tilde{\beta}(t) = e^{-i\int_0^t \omega(t')dt'}, \quad (8)$$

$$\tilde{\gamma}(t) = -i \left(\int_0^t e^{i\int_0^{\tau'} \omega(\tau)d\tau} f(t')dt' \right) e^{-i\int_0^t \omega(t')dt'} \quad (9)$$

In the particular case of constant ω we have

$$z(t) = e^{-i\omega t} \left(z - i \int_0^t e^{i\omega t'} f(t') dt' \right). \quad (10)$$

The forced oscillator system (1) admits [26] linear and analytic in terms of a invariant boson annihilation operator $A_c(t)$,

$$A_c(t) = \beta(t)a + \gamma(t) = U_{cs}(t)aU_{cs}^\dagger(t), \quad (11)$$

where $U_{cs}(t)$ is the unitary evolution operator, corresponding to (1), and

$$\beta(t) = e^{i \int_0^t \omega(t') dt'} = \tilde{\beta}^{-1}(t), \quad \gamma(t) = i \int_0^t f(t') e^{i \int_0^{t'} \omega(\tau) d\tau} dt' = -\tilde{\gamma}(t). \quad (12)$$

For any system (with evolution operator $U(t)$) the time-evolved CS $|z; t\rangle$ are eigenstates of the corresponding invariant annihilation operator $A(t) = U(t)aU^\dagger(t)$ with constant eigenvalues z , $A(t)|z; t\rangle = z|z; t\rangle$, and can be represented in the form of invariantly displaced time-evolved ground state $|0; t\rangle = U(t)|0\rangle$ [21, 12, 26],

$$|z; t\rangle = D(z, A(t)) |0; t\rangle, \quad D(z, A(t)) = e^{A^\dagger(t)z - z^* A(t)}. \quad (13)$$

If $A(t)$ is invariant then $A^\dagger(t)$ also is, and any other combination of them is also invariant. In particular $A^\dagger(t)A(t)$ and $D(z, A(t))$ are also invariant operators of the forced oscillator (1). Invariant operators are very useful, since they transform solutions into solutions, as demonstrated in (13).

The invariant boson ladder operator (11) is a simple particular case of linear invariants of general quadratic quantum system, constructed first in [21, 12]. For the nonstationary quantum oscillator Hermitian quadratic in a and a^\dagger invariant was constructed and studied by Lewis and Riesenfeld [15]. Using these properties of the invariants it was shown [26] that a given Hamiltonian H preserves the temporal stability of CS $|z\rangle$ if and only if it admits invariant of the form $A_c = \beta(t)a + \gamma(t)$. The general form of such Hamiltonian coincides with Glauber-Mehta-Sudarshan coherence Hamiltonian (1).

3 Temporal stability of canonical fermion CS

Fermion coherent states (CS) are defined (see [16, 18, 17, 1, 19, 20, 14, 13, 3]) as eigenstates of the fermion annihilation operator b ,

$$b|\zeta\rangle = \zeta|\zeta\rangle, \quad (14)$$

where the eigenvalue ζ is a Grassmann variable: $\zeta^2 = 0$, $\zeta\zeta^* + \zeta^*\zeta = 0$. Recall the fermion algebra:

$$\{b, b^\dagger\} \equiv bb^\dagger + b^\dagger b = 1, \quad b^2 = b^{\dagger 2} = 0. \quad (15)$$

For definiteness eigenstates of fermion ladder operator b should be called *canonical fermion CS*. This is in analogy to the eigenstates of boson annihilation operator a , which are known as Glauber CS, and *canonical boson CS* as well. In terms of the Grassmannian eigenvalues ζ many of the properties of $|\zeta\rangle$ repeat the corresponding ones of the bosonic CS $|z\rangle$ [3]. In particular one has

$$|\zeta\rangle = D(\zeta)|0\rangle = e^{-\frac{1}{2}\zeta^*\zeta}(|0\rangle - \zeta|1\rangle). \quad (16)$$

$$\int d\zeta^* d\zeta |\zeta\rangle\langle\zeta| = 1, \quad (17)$$

where $D(\zeta) = \exp(b^\dagger\zeta - \zeta^*b)$, $|0\rangle$ is the fermionic vacuum, $b|0\rangle = 0$, and $|1\rangle$ is the one-fermion state, $|1\rangle = b^\dagger|0\rangle$. The integrations over ζ and ζ^* are performed according to the Berezin rules (see e.g. [3])

$$\int d\zeta^* d\zeta \zeta \zeta^* = 1, \quad \int d\zeta^* d\zeta \zeta = \int d\zeta^* d\zeta \zeta^* = \int d\zeta^* d\zeta 1 = 0. \quad (18)$$

The temporal stability of the canonical fermion CS is defined in analogy to the temporal stability of canonical boson CS: the evolution of an initial $|\zeta\rangle$ is stable if the time-evolved state $|\zeta;t\rangle = U(t)|\zeta\rangle$ ($U(t)$ being the evolution operator of the system) remains eigenstate of b in all later time,

$$b|\zeta;t\rangle = \zeta(t)|\zeta;t\rangle. \quad (19)$$

It is clear that the time-evolved states $|\zeta;t\rangle$ also obey the overcompleteness relation (17) and are eigenstates of the invariant ladder operator $B(t) = U(t)bU^\dagger(t)$.

To find the fermion coherence Hamiltonian we first note that the ladder operator invariant $B(t)$ and fermion annihilation operator b should commute since they are supposed to have simultaneously an overcomplete set of eigenstates (we suppose that $\zeta(t)$ and ζ commute). Second, we note that, due to the nilpotency of b and b^\dagger , the general form of a fermionic operator is a (complex) linear combination of b , b^\dagger and $b^\dagger b$. Such a combination will commute with b under certain simple restrictions. Taking then into account that the invariants $B(t)$ and $B^\dagger(t)$ have to obey the fermion algebra (15) we derive that $[b, B(t)] = 0$ if and only if $B(t)$ is proportional to b , $B(t) = \beta'(t)b$. Thus the *fermion coherence Hamiltonian* should admit dynamical invariant of the form (Invariants of other forms for fermion systems have been considered by Dodonov and Man'ko [7], Abe [2], and Cherbal et al [5])

$$B_c(t) = \beta'(t)b, \quad (20)$$

where $\beta'(t) = \exp(i\varphi(t))$, the phase $\varphi(t)$ being arbitrary function of time. As we have already noted at the end of the preceding section, similar form of the ladder operator invariant A_c , eq. (11), is required in the case of boson systems

[26]. To obtain now the general fermion coherence Hamiltonian H_{fc} we apply the defining requirement for quantum time-dependent invariants $B(t)$,

$$\frac{\partial}{\partial t}B(t) - i[B(t), H] = 0, \quad (21)$$

to the operator (20). The general form of fermionic (one-mode) Hamiltonian H_{f} is a Hermitian linear combination of b , b^\dagger and $b^\dagger b$,

$$H_{\text{f}} = \omega'(t)b^\dagger b + f'(t)b^\dagger + f'^*(t)b + g'(t), \quad (22)$$

where $\omega'(t)$ and $g'(t)$ are real functions of time. The substitution of this H_{f} into (21) for $B_c(t)$ produces the two conditions

$$\dot{\beta}' = i\beta'\omega', \quad 0 = \beta'f'. \quad (23)$$

These simple conditions are readily solved,

$$f'(t) = 0, \quad \beta'(t) = \exp\left(i \int_0^t \omega'(\tau) d\tau\right),$$

leading to the Hamiltonian

$$H_{\text{fc}} = \omega'(t)b^\dagger b + g'(t), \quad (24)$$

which is the most general form of fermion coherence Hamiltonian.

Next we find the expression of the eigenvalue $\zeta(t)$ of b ($b|\zeta; t\rangle = \zeta(t)|\zeta; t\rangle$) in terms of the parameter functions $\omega'(t)$, $g'(t)$ in H_{fc} . In this aim we note that if the evolution of an initial CS $|\zeta\rangle$ is governed by H_{fc} one can represent the corresponding time-evolved state $|z; t\rangle_c$ in a form similar to (13),

$$|\zeta; t\rangle_c = D(\zeta, B_c(t))|0; t\rangle = e^{i\varphi(t)}|\zeta(t)\rangle. \quad (25)$$

Then we apply b , $b = \beta'^{-1}B_c(t)$, to $|\zeta; t\rangle$, using the fermion algebra for B_c and B_c^\dagger and the relation $B_c|0; t\rangle = 0$. In this way we arrive to the following simple expression for the eigenvalue $\zeta(t)$,

$$\zeta(t) = \beta'^{-1}(t)\zeta = e^{-i \int_0^t \omega'(\tau) d\tau} \zeta. \quad (26)$$

The results (24) and (26) are similar in form, but not identical, to those for the boson systems (1) and (8). The fermion coherence Hamiltonian (24) is of the form of a *nonforced oscillator* with time dependent frequency (nonstationary fermion oscillator), while the boson coherence Hamiltonian (1) is of the more general form of the nonstationary *forced oscillator*.

The exact evolution of fermion CS $|\zeta; t\rangle$, governed by the more general nonstationary forced oscillator Hamiltonian H_{f} is constructed, using the dynamical invariant ladder operator method [21, 12, 15], in [5],

$$|\zeta; t\rangle = \exp[B^\dagger(t)\zeta - \zeta^* B(t)]|0; t\rangle, \quad (27)$$

where $B(t)$ is the invariant fermion annihilation operator for H_f , and $|0;t\rangle$ is the time-evolved ground state, $B(t)|0;t\rangle = 0$. The invariant $B(t)$ is found in the form

$$B(t) = \nu_-(t)b + \nu_+(t)b^\dagger + \nu_3(t)(b^\dagger b - \frac{1}{2}), \quad (28)$$

where $\nu_\pm(t)$, $\nu_3(t)$ are solutions to the auxiliary system of equations

$$\dot{\nu}_3 = 2i(\nu_+ f'^* - \nu_- f'), \quad (29)$$

$$\dot{\nu}_+ = i(\nu_3 f' - \nu_+ \omega'), \quad (30)$$

$$\dot{\nu}_- = i(\nu_- \omega' - \nu_3 f'^*), \quad (31)$$

subjected to the initial conditions $\nu_-(0) = 1$, $\nu_+(0) = 0$, $\nu_3(0) = 0$. One readily sees that this invariant will be proportional to b (as required by the coherence preserving condition $[b, B(t)] = 0$) iff $\nu_3(t) = 0 = \nu_+(t)$. And eqs. (29) - (31) show that this is possible if $f' = 0$, i.e. if H_f takes the previously obtained form of fermion coherence Hamiltonian H_{fc} , eq. (24).

4 Grassmannian Coherence Hamiltonians

The concepts of stable time evolution of fermion CS leads in a natural way to the Grassmannian Hamiltonian operators of the form

$$H_{gf}(t) = \omega(t)b^\dagger b + \eta(t)b^\dagger - \eta^*(t)b + \delta(t), \quad (32)$$

where $\omega(t)$ and $\delta(t)$ are arbitrary time-dependent real functions and $\eta(t)$ is a Grassmann variable: $\eta\eta^* = -\eta^*\eta$, $\eta^2 = 0$. This is a Grassmannian generalization of the fermion forced oscillator, whose dynamical invariants and CS have been studied in ref. [5]. Here we are going to show that this extension of fermion oscillator preserves the temporal stability of fermion CS. In this aim we first note that if the time evolution $|\zeta;t\rangle$ of an initial (at $t = 0$) state $|\zeta\rangle$ is temporally stable, then it should have the form (see eq.(25)),

$$|\zeta;t\rangle = e^{i\varphi(t)}|\zeta(t)\rangle \equiv |\zeta;t\rangle_c, \quad (33)$$

where $\zeta(0) = \zeta$, and $\varphi(t)$ is a real phase. In particular, the evolution $|0;t\rangle$ of the ground state $|0\rangle$ is stable if

$$|0;t\rangle = e^{i\theta(t)}|0\rangle. \quad (34)$$

It can be readily seen that the time-stable ground state obeys the Schrödinger equation (SE)

$$i\partial_t|0;t\rangle = H_0(t)|0;t\rangle \quad (35)$$

with Hamiltonian

$$H_0(t) = \omega(t)b^\dagger b + \beta(t), \quad (36)$$

where $\omega(t)$ is arbitrary real, and $\beta(t) = \dot{\theta}(t)$. We say that the Hamiltonian $H_0(t)$ (with arbitrary real $\omega(t)$, $\beta(t)$) preserves the fermion ground state stable. It is not difficult to prove that this is the most general form of Hamiltonians that preserve the stability of the ground state.

Next we recall the well known result that if $|\psi_0\rangle$ obeys the SE with H_0 then the unitarily transformed state $|\psi_1\rangle = U|\psi_0\rangle$ obeys the SE with Hamiltonian $UH_0U^\dagger - iU\partial_t U^\dagger$. This means that the temporally stable CS $\exp(i\varphi(t))|\zeta(t)\rangle$ satisfies the SE with Hamiltonian

$$H_{\text{gf}}(t) = D(\zeta(t), b)H_0(t)D^\dagger(\zeta(t), b) - iD(\zeta(t), b)\frac{\partial}{\partial t}D^\dagger(\zeta(t), b) \quad (37)$$

where $D(\zeta(t), b) = \exp(b^\dagger\zeta(t) - \zeta^*(t)b)$. Using the properties of the displacement operator (recalling that $b\zeta = -\zeta b$, $b\zeta^* = -\zeta^*b$ [3]),

$$D(\zeta, b)bD^\dagger(\zeta, b) = b - \zeta, \quad D^\dagger(\zeta, b)bD(\zeta, b) = b + \zeta,$$

we find

$$DH_0D^\dagger = \omega(t)(b^\dagger - \zeta^*)(b - \zeta) + \beta(t), \quad (38)$$

$$D\frac{\partial}{\partial t}D^\dagger = \dot{\zeta}^*b + \dot{\zeta}b^\dagger + \frac{1}{2}(\zeta^*\dot{\zeta} - \dot{\zeta}^*\zeta), \quad (39)$$

$$\begin{aligned} H_{\text{gf}}(t) &= \omega b^\dagger b + \left(\omega\zeta - i\dot{\zeta}\right)b^\dagger - \left(\omega\zeta^* + i\dot{\zeta}^*\right)b \\ &\quad + \beta + \omega\zeta^*\zeta - \frac{i}{2}(\zeta^*\dot{\zeta} - \dot{\zeta}^*\zeta). \end{aligned} \quad (40)$$

By identification of the two expressions of $H_{\text{gf}}(t)$ given respectively in eqs. (32) and (??), one obtains the relations between the corresponding parameter functions:

$$\eta = \omega\zeta - i\dot{\zeta}, \quad (41)$$

$$\delta(t) = \beta + \omega\zeta^*\zeta - \frac{i}{2}(\zeta^*\dot{\zeta} - \dot{\zeta}^*\zeta). \quad (42)$$

The above equations shows that if there is a set of eigenstates $|\zeta; t\rangle$ of b with eigenvalues $\zeta(t)$, $\zeta(0) = \zeta$, and the ground state obeys SE with H_0 , eq. (36), then $|\zeta; t\rangle$ obeys the SE with H_{gf} , i.e. the time evolution of initial $|\zeta\rangle$ governed by H_{gf} is temporally stable. The Grassmannian coefficients $\eta(t)$, $\delta(t)$ are determined by the given $\zeta(t)$ according to eqs. (41) and (42), $\omega(t)$ and $\beta(t)$ being arbitrary real.

And the inverse is also true: the Grassmannian Hamiltonian H_{gf} , eq. (32), preserves the temporal stability of eigenstates $|\zeta\rangle$ of b (that is fermion CS), the eigenvalues $\zeta(t)$ being determined by the "classical equation" (following from (41))

$$i\dot{\zeta} = \omega\zeta - \eta. \quad (43)$$

The stable time evolved CS is $\exp(i\varphi(t))D(\zeta(t), b)|0\rangle$, the phase $\varphi(t)$ being determined by the equation (following from (36), (42) and (41))

$$\dot{\varphi} = \delta - \frac{1}{2}(\zeta^*\eta + \eta^*\zeta). \quad (44)$$

Note that in derivation of (43), (44) we have presupposed that the Grassmann variables η and ζ anticommute.

Thus the Grassmannian fermionic forced oscillator, eq. (32), is fermion coherence Hamiltonian, too. There is only one possibility to restrict oneself with ordinary fermion coherence Hamiltonian, i.e. with ordinary complex coefficient η and real δ in (32): this is, as it follows from eqs. (41) and (42), to put $\eta = 0$. Then one returns to H_{fc} , eq. (24).

Concluding Remarks

In this article, we have extended the earlier results of the *boson* canonical coherence Hamiltonian [11, 23] and boson invariant ladder operators [26] to the *fermion* coherence Hamiltonian and fermion invariant ladder operators. The fermion coherence Hamiltonian is obtained in the form of nonstationary *non-forced* oscillator. As an expression in terms of the ladder operators, this form is more restricted than the corresponding expression of the boson coherence Hamiltonian, which is of the form of nonstationary *forced* oscillator. This more particular form is mainly due to the nilpotency of the fermionic annihilation and creation operators. The nilpotent property, and the related anticommutation relations, of fermion ladder operators lead to the very simple form of the general (one mode) fermion Hamiltonian, namely to the form of forced fermionic oscillator, which is a general element of the simple algebra of $SU(2)$. Accordingly, the fermion coherence Hamiltonian is a particular element of $su(2)$ algebra; it is proportional (up to an additive C -number term) to the third generator of $SU(2)$. The symmetry of the bosonic coherence Hamiltonian is quite different; it is a general element of the nonsimple oscillator algebra.

We have finally shown that the parallel between the boson coherence Hamiltonian and the fermion coherence Hamiltonian can be formally restored if one admits Grassmann variables as Hamiltonian parameters: then the fermion coherence Hamiltonian takes again the form of (Grassmannian) forced oscillator.

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